



## Claycots School calculation policy

This calculation policy has been written to be used alongside the National Curriculum (2014). It is important that the children consolidate their year group stage, before they move onto more challenging concepts. Mathematical understanding is developed through the CPA approach.

- Concrete representations such as: dienes, place value counters, Numicon & Cuisenaire rods.
- Pictorial representations such as: bar models & part-whole models
- Abstract representations such as: column addition & subtraction, multiplication & division.

Children should be taught to use mental maths whenever possible, and this should be taught explicitly. They should not be using written methods for simple calculations; we must work towards developing the children's efficiency in Maths. The main aim of this policy is to ensure consistency and allows for the children to develop a deep and sustainable understanding of Maths.

This calculation policy should be used alongside planning to ensure that there is consistency within and across year groups. If children are finding difficulty or making a significant amount of errors, then they should return to the previous stage. Teacher's assessment should be used to identify the children's next step.



## Addition & Subtraction

### EYFS

**Children learn through play – activities presented should be engaging and creative.**

### Addition

Before addition can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting, number order and number recognition through practical activities and games. This is taught through both adult led and child-initiated games. Children also learn to count 1-1 (pointing to each object as they count) and also learn that anything can be counted, e.g. claps, steps and jumps. This is reinforced by opportunities provided in the outdoor area for the children to count. E.g. counting building blocks, sticks, rocks etc.

In Reception, before addition can be introduced, children build on concepts taught in Nursery. Children need to have a secure knowledge of number in order to begin addition. Children are then introduced to the concept of addition through practical games and activities. Children act out addition calculations to physically add two groups of objects together and use arm gestures to represent the signs  $+$  and  $=$ . This is reinforced by opportunities provided in the outdoor area for the children i.e. adding together groups of building blocks, sticks etc. Children build on their previous knowledge of 'more' by learning that adding two groups of objects together gives them a larger number (more objects). Adults model addition vocabulary supported by age appropriate definition. An example of this is "addition means we add two groups together / we put 2 lots of objects together. Equals means we find out how many we have got altogether. 3 add 2 equals 5! We have got 5 altogether". Adults support children in recording their addition sums in the written form on whiteboards and in their maths books

**\*To add pictures of engaging ways to understand numbers & to add\***




## Subtraction


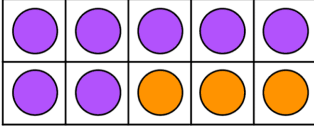




Before subtraction can be introduced, children need to have a secure knowledge of number. In Nursery, children are introduced to the concept of counting backwards. This is taught through child-initiated games indoors and outdoors such as acting out counting songs and running races (children shouting “5,4,3,2,1,0 - GO!”).



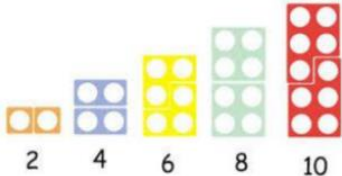
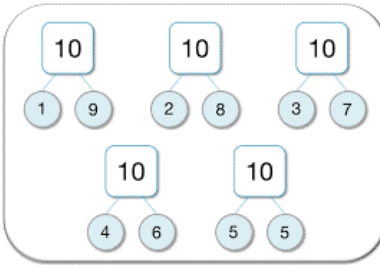
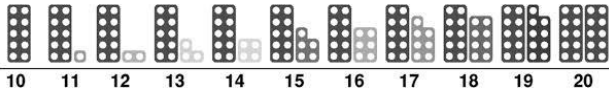
In Reception, before subtraction can be introduced, children build on concepts taught in Nursery. Children need to have a secure knowledge of number in order to begin subtraction. Children are then introduced to the concept of subtraction through practical games and activities. Children act out subtractions to physically subtract a number of objects from a group. Children use arm gestures to represent the signs - and =. This is reinforced by opportunities provided in the outdoor area for the children to count e.g. counting building blocks, twigs etc. Children build on their previous knowledge of ‘less’ by learning that subtracting means taking away a certain number of objects from a group (leaving them with less objects). Adults model subtraction vocabulary supported by age appropriate definition. An example of this is “subtraction means we take away objects from a group.” Equals means we find out how many we have got left. Wow! We have only got 3 left!” Adults support children in recording their subtractions in the written form on whiteboards and in their maths books

**\*To add pictures of engaging ways to understand numbers & to subtract\***


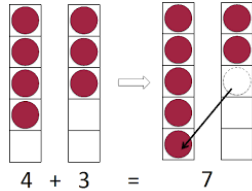



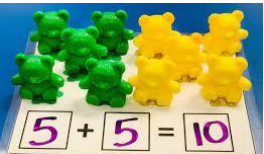
Early learning goals	Concrete	Pictorial	Abstract
Have a deep understanding of numbers to 10, including the composition of each number;	The example below demonstrates how the Numicon shapes can be used to show that 3 and 4 make 7... This can be used for numbers up to 10. The children can also place the numicon on top of each other.	When counting, pictures can be used to count 	Here children can have the digit in number form or written form.  E.g. $6 + 4 =$


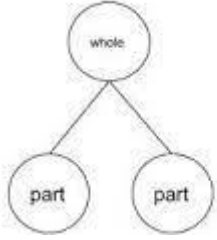
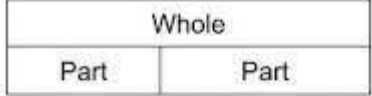
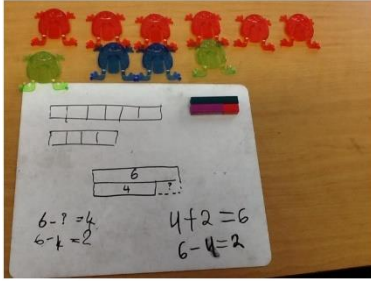
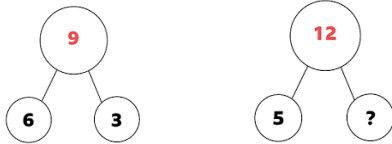
	 <p>Cubes can also be used here to show relationships in numbers up to 10. *Add image</p>	<p>Ten frames can also be used here</p>  <p>*Add further examples of composition of numbers with a ten frame</p>	<p><math>5 + 5 =</math></p> <p>Six plus four equals</p> <p>Five plus five equals</p> <p>In how many ways can you make the number X.</p>
<p>Subitise (recognise quantities without counting) up to 5;</p>	<p>Numicon pieces could be used:</p>  <p>Cubes:</p>  <p>Ten frames:</p>	<p>Dots to represent numbers</p>  <p>Use of ten frames:</p> 	<p>Here number cards can be shown</p>

			
Automatically recall (without reference to rhymes, counting or other aids) number bonds up to 5 (including subtraction facts) and some number bonds to 10, including double facts.	<p>Here we can see how Numicon can be used to find number bonds to make 10</p>  <p><i>Ten frames can also be used</i></p> <p>Double facts:</p> 	<p>Use of numicon &amp; ten frames to support number bonds to 5 and 10</p> <p>Use of numicon &amp; ten frames to identify double facts</p> <p><i>Pictorial to be used alongside concrete shown.</i></p> 	$3 + \_ = 5$
Verbally count beyond 20, recognising the pattern of the counting system;	<p>Ten frames</p> <p>Numicon</p>  <p>- Create similar example beyond 20 of above</p>	<i>Pictorial to be used alongside concrete shown.</i>	Use regular opportunities to count beyond 20 – forwards and backwards
Compare quantities up to 10 in	Coloured bears, cubes, lego, beads, buttons, small toys, pasta etc	<i>Pictorial to be used alongside concrete shown.</i>	Give children two quantities to

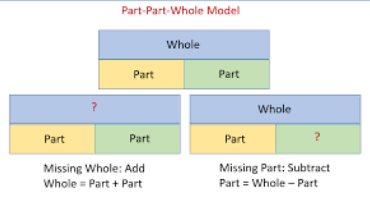



different contexts, recognising when one quantity is greater than, less than or the same as the other quantity;			compare – encourage use of key vocabulary – greater than, less than, equal to
Explore and represent patterns within numbers up to 10, including evens and odds, double facts and how quantities can be distributed equally.	<p>Ten frames</p> <ul style="list-style-type: none"> <li>- Use concrete resource alongside pictorial Numicon</li> <li>- Use concrete resource alongside pictorial (can be arranged in same way as ten frame examples</li> </ul> <p><i>For odd vs. even – to talk about how the pieces are arranged etc</i></p>	 	<p>Abstract alongside pictorial and concrete.</p> <p>E.g. <math>2 + 2 =</math>  <math>4 + 4 =</math>          (add a non-double i.e. <math>3 + 4 =</math>)</p> <p>What do you notice?</p>
Adding and subtracting	 	<p><b>Use of coloured bears</b></p> <p><b>Use of multilink</b></p>	 <p><math>3 + 2 = 5</math></p> <p>I have 3 sweets and you have 2 sweets. How many sweets do we have altogether?  <b><math>3 + 2 =</math></b></p> <p>If I have 4 counters and add 3 more, how many do I have altogether?</p>

	<p><b>The children will build the 7 cubes, take away 3 cubes and then count the remaining cubes.</b></p> <p>Here are 7 cubes. Take away 3 cubes.</p> 	 <p>4 + 3 = 7</p> <p>6 - 2 = 4</p>  <p><b>Pictures of familiar items can be used at the pictorial stage to represent the quantity.</b></p>	<p><b>4+3 =</b></p> <p>If you have six apples and take two away, how many are left?</p> <p><b>6-2 =</b></p>
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<p><b>Year 1</b></p> <p><i>Main formal method – progression through use of a number line</i></p>			
Objectives	Concrete	Pictorial	Abstract
<p>read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs</p>	<p>Use of same resources as EYFS – Inc. dienes, counters, bear etc</p> <p>E.g.</p> 	<p><i>In the first instance children can use same picture representation as concrete.</i></p> <p>Move onto use of a part – part model</p> <p>Later, move onto use of bar model</p> <p>Children will need a thorough explanation of representations</p>	<p>5 + 5 =</p> <p>7 – 3 =</p> <p>Introduce children to key vocabulary when adding and subtracting</p> <p>You could also begin to vary to position of the equal sign later in the year to develop</p>

	<p>Here are 7 cubes. Take away 3 cubes.</p> 	  <p>Bar model</p>  <p>Children can also use Cuisenaire rods. This will then encourage the children to think about the size of the number in comparison to the other.</p>	<p>the children's understanding of the meaning of the = sign</p>
Represent and use number bonds and related subtraction facts within 20	<p>Ten frames</p> <p>Numicon</p> <p>Dienes</p>	<p>Part whole models</p>  <p>Bar model</p>	<p><math>6 + 4 =</math></p> <p><math>16 + 4 =</math></p> <p><math>8 + 2 =</math></p> <p><math>18 + 2 =</math></p>



			
<p>Add and subtract one-digit and two-digit numbers to 20, including zero</p>	<p>Firstly, children need to use objects to count how many there are altogether. Then they should arrange the objects in a line (each number), count them and then add them together.</p> <p><math>6 + 3 =</math> (inc. pic using cubes)</p> <p>Children should use concrete objects to take away.</p> <p>Encourage children to use objects show the amount and physically take away the number they are subtracting.</p> <p><math>12 - 4 =</math> (Inc. pic using cubes)</p>	<p><b>Addition</b> Use a number line to count on, starting from the largest number</p>  <p><math>6 + 3 = 9</math> Start at the 6 and use your finger to count on (forward).</p> <p>Then use a marked number line: <math>6 + 6 = 12</math></p>  <p>Put your finger on 6 and count on 6 to 12.</p> <p><math>8 + 7 =</math></p> <p>+1 +1 +1 +1 +1 +1 +1</p>  <p>Put your finger on 8 and count on 7.</p>	<p><i>Abstract shown alongside pictorial</i></p>

Then children can continue to practise this with an unmarked number line.

### **Subtraction**

Children should draw objects/ or have them available to them and cross out the objects to find out how any are left

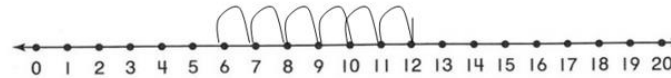
To practise counting back from a given number, children should use a number track at the beginning to help them.



$7 - 3 =$  put your finger on the 7 and count back 3.'

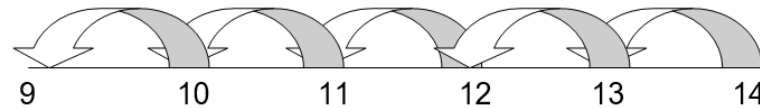
Then move onto a marked number line.

$$12 - 6 = 6$$



Put your finger on the 12 and count back 6.

Children will then move onto an unmarked number line – make sure that the children are confident using a marked number line first.



Put your finger on 14 and count back 5.

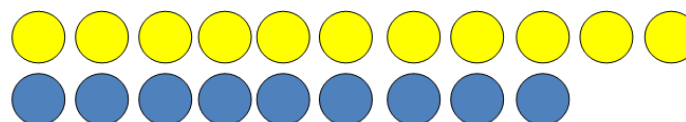
solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = - 9$ .

Chn to use ten frames and counters to support in solving missing number problems

Then teach the children to count on to find small differences between two numbers. It is important that this method is not used for big differences but it is important for children to understand the term 'difference'.

Count up from the smallest to the largest number using resources for examples cubes, dienes, counters, number lines.

$$11 - 9 = 2$$

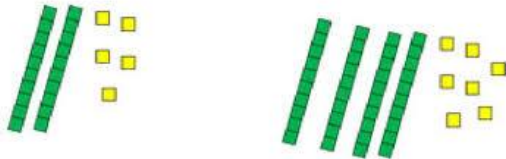
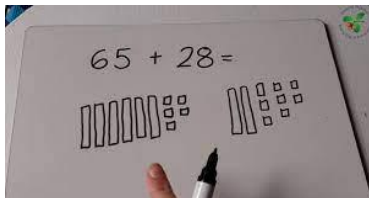


'The difference between 9 and 11 is 2. Count up from 9 to 11.

Remember if the children consistently make errors, then you should take them back to the previous stage.

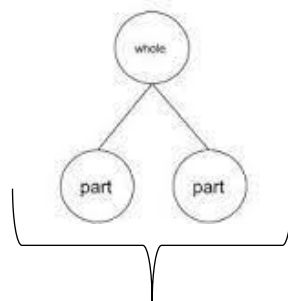
Use concrete and pictorial representations to solve missing number problems:

<p> <math>10 = 4 + 6</math>  <math>10 = 6 + 4</math>  <math>6 = 10 - 4</math>  <math>4 = 10 - 6</math> </p> <p>Tens Frame</p>	<p> <math>6 + 4 = 10</math>  <math>4 + 6 = 10</math>  <math>10 - 4 = 6</math>  <math>10 - 6 = 4</math> </p> <p>Part Whole Model</p>	<table border="1"> <tr> <td colspan="2">10</td> </tr> <tr> <td>6</td> <td>4</td> </tr> </table> <p> <math>6 + 4 = 10</math>  <math>4 + 6 = 10</math>  <math>10 - 4 = 6</math>  <math>10 - 6 = 4</math> </p> <p>Bar Model</p>	10		6	4
10						
6	4					

Year 2								
Objectives	Concrete	Pictorial	Abstract					
Solve problems with addition & subtraction	<p>Children to use concrete resources to add amounts.</p> <p>Where children are moving onto larger numbers, some of the concrete resources used in Y1 – will be replaced for place value counters and dienes. This should be modelled to the children – this will begin to introduce the concept of place value and it is important to highlight why the resources are being used.</p> <p>Children to be able to represent amounts using dienes. E.g. <math>25 + 47 =</math></p> <div></div> <p>Where an exchange is needed, children to physically exchange 10 ones for a 10 rod.</p> <table border="1" data-bbox="463 737 1104 809"><tr><td colspan="2">72</td></tr><tr><td>25</td><td>47</td></tr></table> <p>Children can then move onto showing the dienes in their books.</p> <div data-bbox="486 976 853 1174"></div>		72		25	47	<p>When subtracting, the children should show the total using dienes and physically remove the amount they are taking away to reveal the answer. <b>*To add image</b></p>	As shown
72								
25	47							

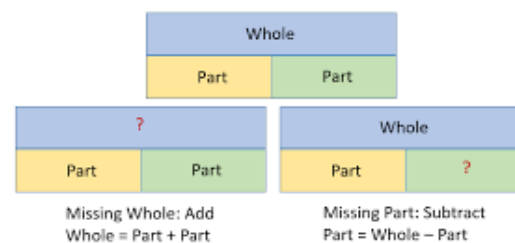
Children will continue to use the part whole model and the bar model to represent their calculations and will be familiar with this due to exposure in Yr1.

### Part whole model



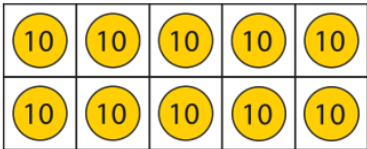
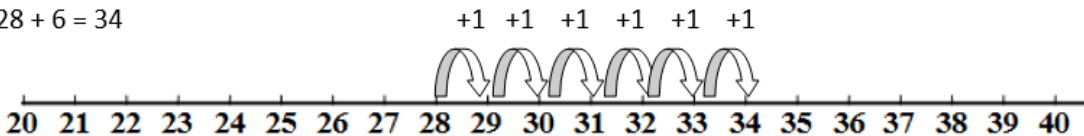
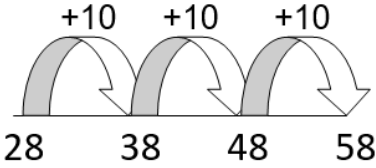
Each part adds together to make the whole.

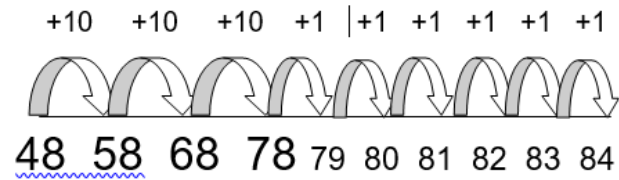
### Bar model



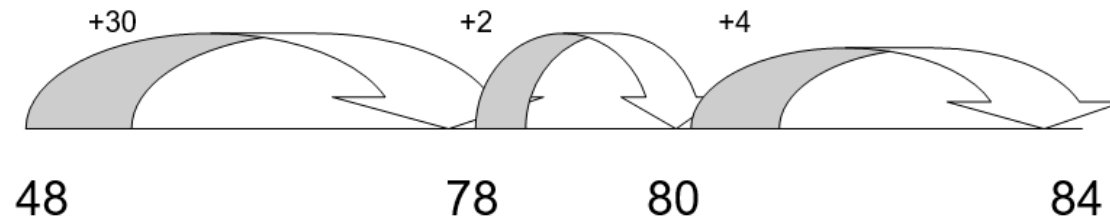
Recall and use addition and subtraction facts to 20 fluently, and

Recap use of ten frames to show number bonds to 10 and 20. Numicon can also be used here. Children will then move onto use of ten frames to represent 100.  
E.g.

<p>derive and use related facts up to 100</p>		<p>Children will understand the use of a ten frame and be familiar with each part representing one, the children can then use this to solve number bonds to 100.</p>  <p><i>Add further examples of use of concrete resources to support number sense to 20 to apply to up to 100</i></p>	
<p>add and subtract numbers using concrete objects, pictorial representations, and mentally, including:</p> <ul style="list-style-type: none"> <li>- a two-digit number and ones</li> <li>- a two-digit number and tens</li> <li>- two two-digit numbers</li> <li>- adding three one-digit numbers</li> </ul>		<p>Counting on, on an empty number line within 100.</p> <p><math>28 + 6 = 34</math></p>  <p>And in tens.</p> <p><math>28 + 30 = 58</math></p>  <p>You should use a number square alongside to show the jumps in 10.</p> <p><math>48 + 36 = 84</math></p> <p>You should put the biggest number first (48) and then partition the smaller number (<math>36 = 30 + 6</math>) and count on: <math>48 + 30 + 6</math></p>	



When the children are ready, they can use more efficient steps  $+30 + 2 + 4$



Using partitioning to add two-digit numbers together.

$$\begin{array}{r} 43 + 25 = 68 \\ \swarrow \downarrow \searrow \swarrow \\ 40 \quad 3 \quad 20 \quad 5 \end{array}$$

$$40 + 20 = 60$$

$$3 + 5 = 8$$

$$60 + 8 = 68$$

Partition the number into tens and ones. Add the tens together and then the ones together and then recombine the two to get the answer.

Then move onto calculations that bridge the tens.

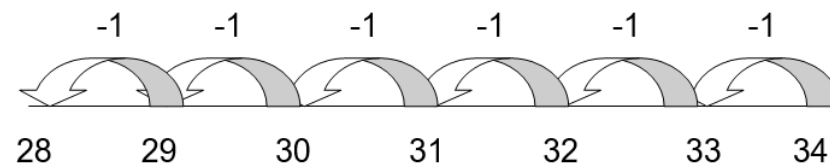
$$\begin{aligned}
 48 + 36 &= 40 + 8 + 30 + 6 \\
 40 + 30 &= 70 \\
 8 + 6 &= 14 \\
 70 + 14 &= 84 \\
 48 + 36 &= 84
 \end{aligned}$$

This is a different way to record the partitioning method. You can further develop by using a 200-number square and calculations that bridge 100.

### ***Subtraction***

Counting back within 100 on an unmarked number line in ones.

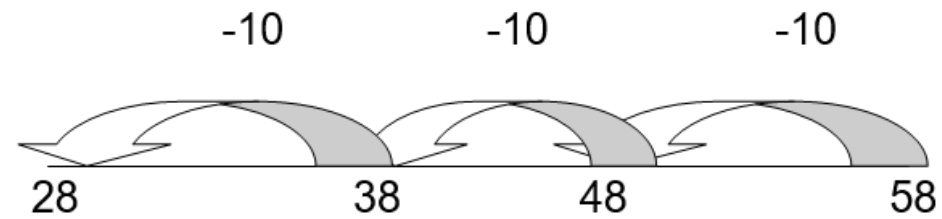
$$34 - 6 = 28$$



### **Then in tens**

$$58 - 30 = 28$$

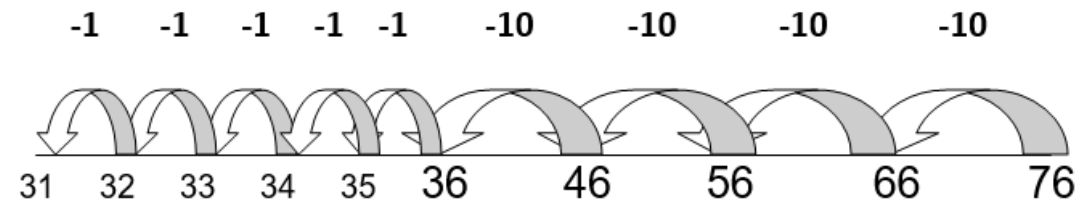




Use in conjunction with 100 number square to show the jumps of ten

Subtraction, using partitioning on an empty number line.

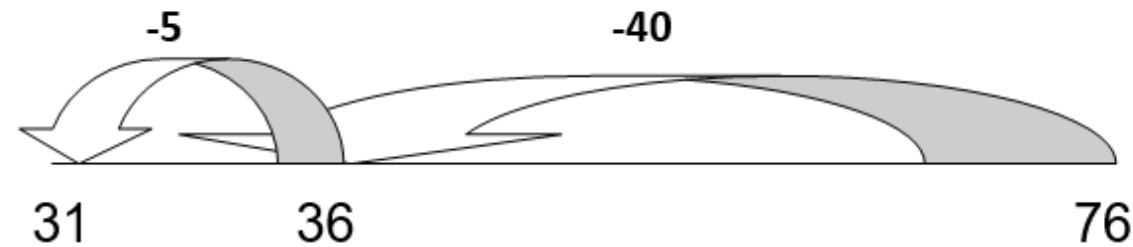
$$76 - 45 = 31$$



Use in conjunction with a 100-number square to show jumps of tens and ones.

If children are confident with this method, show the children how they can subtract more efficiently.

$$76 - 45 = 31$$



Use in conjunction with a 100 square to show jumps of tens and ones.

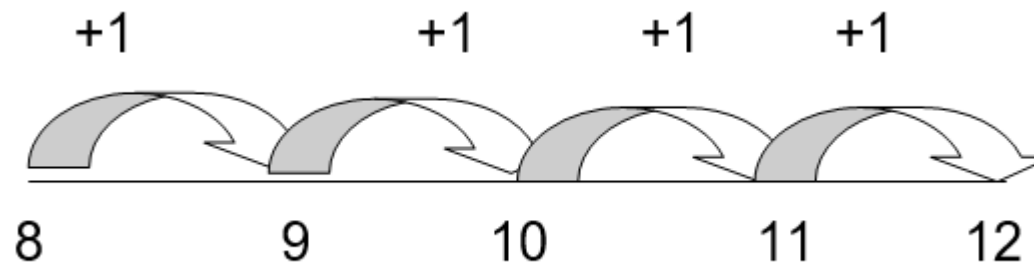
Counting on to find a small difference

See Y1 guidance to help to understand the term 'difference'.

Count up from the smallest number to the largest number to find the difference.

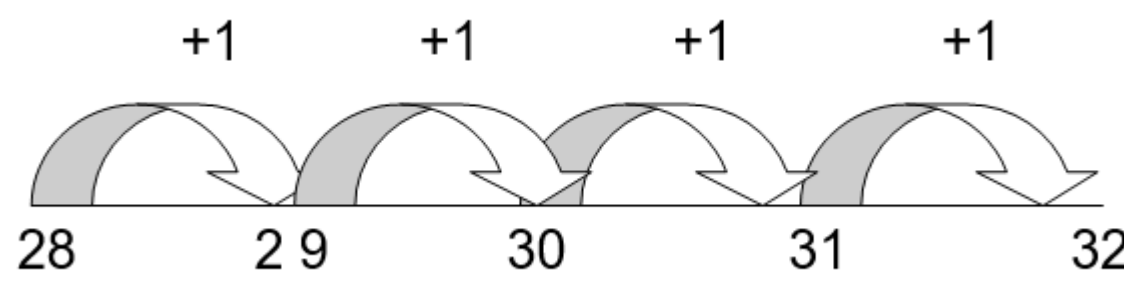
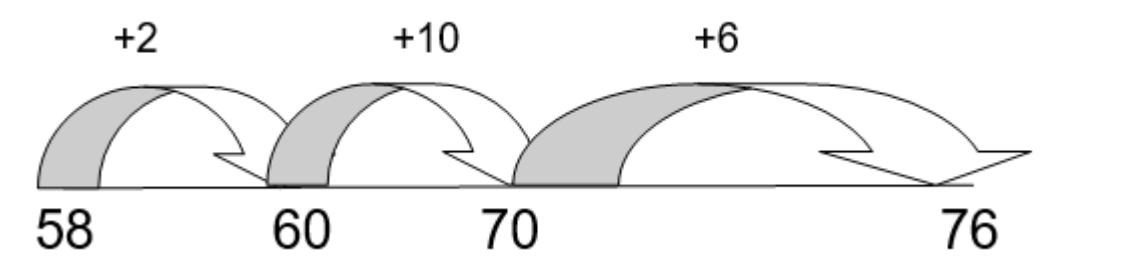
See Year 1 guidance to help understand the term 'difference'.

$$12 - 8 = 4$$



Count up from 8 to 12 The difference between 8 and 12 is 4.

$$32 - 28 = 4$$

		<div data-bbox="649 207 1769 494">  </div> <p>The difference between 28 and 32 is 4.</p> <p>If the children are confident with this method, then you can develop.  <math>76 - 58 = 18</math></p> <div data-bbox="649 750 1769 1021">  </div> <p>'The difference between 58 and 76 is 18. Count up from 58 to 76.'</p> <p>You can develop using subtraction that bridges 100 and you can support this using a 200 number square. If the children are struggling, then take them back to a previous stage.</p>	
<p>Show that addition of two numbers can be done in any order</p>		<p>Addition: Children to use strategies outlined above and to manipulate starting number to highlight commutative law.</p> <p>Subtraction: To use concrete resources to show why subtraction is not commutative.</p>	

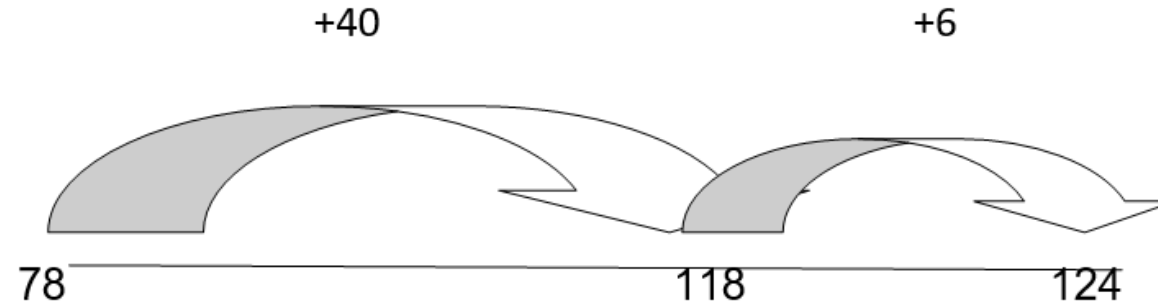


(commutative) and subtraction of one number from another cannot			
-Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.			

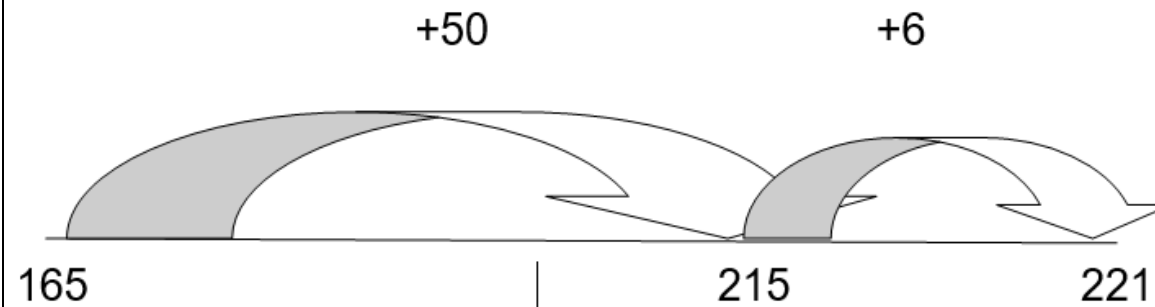
Year 3			
Objectives	Concrete	Pictorial	Abstract
add and subtract numbers mentally, including: <ul style="list-style-type: none"> <li>- a three-digit number and ones</li> <li>- a three-digit number and tens</li> <li>- a three-digit number and hundreds</li> </ul>	<b>Addition</b>  Continue to use the number line for calculations that bridge 100.		
add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction			
estimate the answer to a calculation and use inverse operations to check answers			

solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.

$$78 + 46 = 124$$



Use a 200-number square for counting in tens and bridging over 100.



Use partitioning to add a three digit number and two digit number

$$85 + 37 = 80 + 5 + 30 + 7$$

$$80 + 30 = 110$$

$$5 + 7 = 12$$

$$110 + 12 = 122$$

$$85 + 37 = 122$$

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Then move onto the expanded formal method, both vertically and horizontally. Start with calculations that do not bridge 10 or 100.

$$63 + 32 = 95$$

$$\begin{array}{r} 60 + 3 \\ + 30 + 2 \\ \hline 90 + 5 = 95 \end{array}$$



'Partition the numbers into tens and ones. Add the tens together and then add the ones together. Recombine to give the answer.'

Then...

$$\begin{array}{r} 63 \\ + 32 \\ \hline 5 \text{ (3 + 2)} \\ + 90 \text{ (60 + 30)} \\ \hline 95 \end{array}$$



'Add the ones together first and then the tens in preparation for the formal written method.'

Then...

$$\begin{array}{r} 68 \\ + 24 \\ \hline 12 \text{ (8 + 4)} \\ + 80 \text{ (60 + 20)} \\ \hline 92 \end{array}$$



'Add the ones together first and then the tens in preparation for the formal written method.'

If children are ready, then move onto adding where it is necessary to carry tens from the ones to the tens

$$\begin{array}{r} 68 \\ + 24 \\ \hline 92 \\ 1 \end{array}$$



'Use the language of place value to ensure understanding: Eight add four equals 12. Write two in the ones column and 'carry' one ten across to the tens column. 60 add 20 and the additional ten 'carried' equals 90. Write 9 (9 tens) in the tens column. 92 is the answer...

The digit that has been carried should be recorded under the line in the correct column. When children are ready, calculate numbers that bridge over the ten and hundred.



$$76 + 47 = 123$$

$$\begin{array}{r} 70 + 6 \\ + 40 + 7 \\ \hline 110 + 13 = 123 \end{array}$$



Partition the numbers into tens and ones. Add the tens together and then add the ones together. Recombine to give the answer.

Then...

$$\begin{array}{r} 76 \\ + 47 \\ \hline 13 \text{ (7 + 6)} \\ 110 \text{ (70 + 40)} \\ \hline 123 \end{array}$$



Add the ones first and then the tens in preparation for the formal written method

Then when the children are ready they should bridge over the 100.

$$76 + 47 = 123$$

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ \text{1} \quad \text{1} \end{array} \quad \Rightarrow$$

Use the language of place value to ensure understanding: 'seven add six equals 13. Write three in the ones column and carry one ten across into the tens column. 40 plus 70 and the carried 10 equals 120. Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).

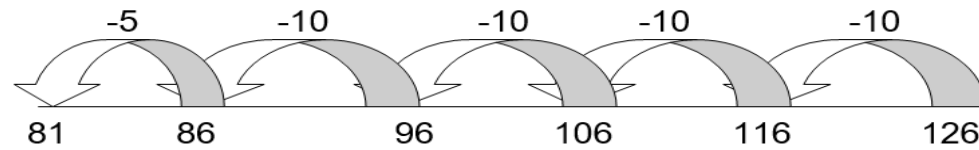
When the children are ready, they can add a three digit to a two-digit number.

$$178 + 43 = 221$$

$$\begin{array}{r} 178 \\ + 43 \\ \hline 221 \\ \text{1} \quad \text{1} \end{array}$$

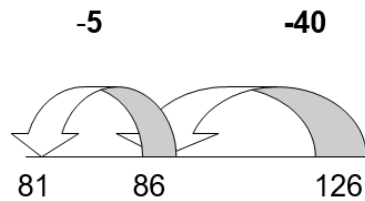
### **Subtraction**

Children should continue to use an empty number line to solve calculations that bridge 100.  
 $126 - 45 = 81$



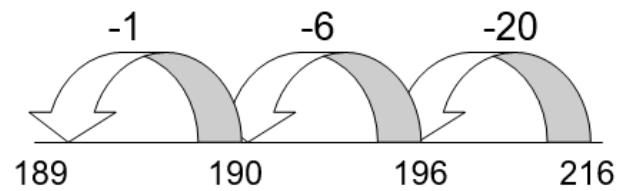
Children can use 200 number square to count back in 10s and bridge 100.

Then the children should learn more efficient steps.

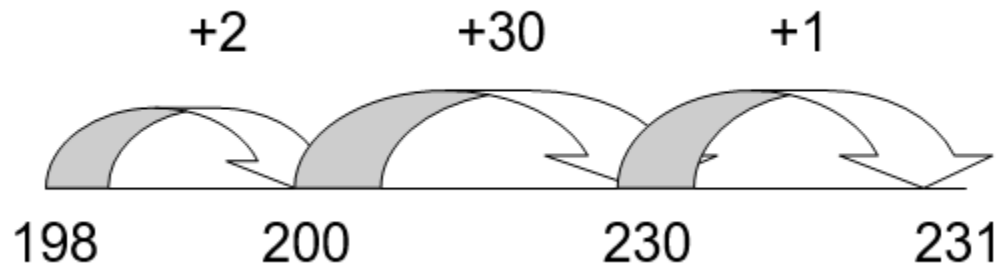


Extend children by using larger numbers to count back.

$$216 - 27 = 189$$



And then by counting on to find a small difference.



$$231 - 198 = 33$$

'The difference between 231 and 198 is 33'

Introduce the expanded written method presented horizontally and vertically. Two digits number should be used when this method is introduced.

$$78 - 23$$

70	8
- 20	3
40	5

$$= 45$$

Partition numbers into tens and ones. Subtract the ones and then the tens. Recombine the numbers for the answer. Children should not need to use decomposition (exchanging) when they are using this method.

You might need to change the '+' for 'and' to avoid confusion.

This will lead into the formal written method.

$$\begin{array}{r} 78 \\ - 23 \\ \hline 55 \end{array}$$

Remember to use the correct language for example eight subtract three and seventy subtract twenty.



You could use a number line for this calculation but use two digit numbers to show this method at the beginning.

Introduce the expanded subtraction where decomposition/ exchange is required.

$$73 - 27 = 45$$

60	
70	13
- 20	7
40	5

Children need to practise partitioning like this and can use resources such as Base Ten to help. When children are confident partitioning like this then introduce the formal written method, involving decomposition/exchange.

$$73 - 27 = 46$$

$$\begin{array}{r} 6 \ 13 \\ 7 \ 3 \\ - 2 \ 7 \\ \hline 4 \ 6 \end{array}$$

Ensure that you use place value language so that the children are clear what is happening in the process. For example, we can't subtract 3 from 7 so we need to exchange a ten for ten ones to give us 60 + 13.

Children should use Base Ten to help understand this concept.

If children are confident using this method, then they can move on to subtracting with three-digit number.

$$235 - 127 = 108$$

$$\begin{array}{r} 2 \ 3 \ 5 \\ - 1 \ 2 \ 7 \\ \hline 1 \ 0 \ 8 \end{array}$$

Use place value language to help the children's understanding.

In this example, it is only necessary to exchange from the tens column. Base Ten can be used to support children's understanding.



If children are making errors with this method then take them back to the previous stage.

Year 4			
Objectives	Concrete	Pictorial	Abstract
add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate	<p><b>Addition</b></p> <p>Revisit the expanded method</p> $176 + 147 = 323$ $  \begin{array}{r}  176 \\  + 147 \\  \hline  13 \quad (7 + 6) \\  110 \quad (70 + 40) \\  200 \quad (100 + 100) \\  \hline  323  \end{array}  $ <p>Then continue with the formal method.</p>		
estimate and use inverse operations to check answers to a calculation			
solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why			

$$176 + 147 = 323$$

$$\begin{array}{r} 147 \\ + 176 \\ \hline 323 \\ \text{1} \quad \text{1} \end{array}$$



'Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one across into the tens column (10). 40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and 'carry' 1 across into the hundreds column (100). 100 add 100 and the 100 that has been carried equals 300. Write 3 in the hundreds column (300).

If the children are confident, then continue to add 4 digits to 3 digits.

$$1845 + 526 = 2371$$

$$\begin{array}{r} 1845 \\ + 526 \\ \hline 2371 \end{array}$$

### **Subtraction**

Ensure that children are confident with the previous methods including using empty number lines for three and four digits when it is appropriate. Also, use the formal method by revisiting the expanded method, if necessary. Base Ten should be used to support the children when required.

$$73 - 27 = 45$$

60	
<del>70</del>	<del>13</del>
- 20	7
40	5

This then leads onto decomposition.

$$\begin{array}{r} 1 \quad 1 \\ 258 \\ - 73 \\ \hline \end{array}$$

$$175$$

In this example, it has been necessary to exchange from the hundreds.

Then move on to subtracting a three-digit number.

$$637 - 252 = 385$$

500		
<del>600</del>	<del>130</del>	7
200	50	2
300	80	5

Children need to be confident partitioning three-digit numbers before moving on to the formal method.





$$\begin{array}{r} 5 \text{ } 13 \\ 637 \\ - 252 \\ \hline 385 \end{array}$$

Use place value language to help children understand the concept and use Base Ten when required.

If children are confident then you can expand to four digit numbers and decimals in the context of money and measure. Remember to return children to the previous stage if they are struggling.

Year 5			
Objectives	Concrete	Pictorial	Abstract
add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction)	<p>Continue to use empty number lines to add large numbers and decimals when required. Continue to develop the use of the formal method adding four and more digits.</p> <p>If children are struggling with a stage then they should go back to the previous stage that they understand.</p> $21848 + 1523 = 23371$ $\begin{array}{r} 21848 \\ + 1523 \\ \hline 23371 \\ \text{1} \quad \text{1} \end{array}$ <p>Use the formal method for the addition of decimal numbers. Ensure that the decimals line up!</p>		
add and subtract numbers mentally with increasingly large numbers			
use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy			
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.			

£154.75 + £233.82 :

$$\begin{array}{r} 154.75 \\ + 233.82 \\ \hline 388.57 \\ 1 \end{array}$$



Continue to use the language of place value to ensure understanding.

### **Subtraction**

Ensure that children are confident in the previous stages before moving on to the next stage. Continue to use empty number lines with larger numbers and decimals when necessary.

Continue to practise using a formal method for three- and four-digit numbers (See Year 4) and return to the expanded method if required.

In this example, the number has to be partitioned into 400 + 90 + 13 to do the calculation.

There are no tens so we have to exchange one hundred for 10 tens before we can exchange a ten for 10 ones.

$$503 - 278 = 225$$

$$\begin{array}{r} 500 + 0 + 3 \\ - 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

$$\begin{array}{r} \phantom{4} \phantom{9} \phantom{13} \\ \cancel{5} \cancel{0} \cancel{3} \\ - 278 \\ \hline 225 \end{array}$$

Then you move to the formal method. There is a high chance that the children will make errors using this method. You could discuss with the children the effectiveness of a mental method for example, would a blank number line be more effective in this situation?

When the children are more confident extend with larger numbers and decimals numbers. Remember to return to an expanded method if the children are struggling.

$$\begin{array}{r}
 \phantom{1}6 \phantom{12} \phantom{11} \\
 12731 \\
 - 1367 \\
 \hline
 11364
 \end{array}$$

In this example, you need to exchange the hundreds and the tens.

If children are struggling with this method, then they should do calculations where there is only one exchange.

Introduce subtraction of decimals, in the context of money and measure.

$$£166.25 - £83.72 = £82.53$$

$$\begin{array}{r}
 \phantom{1}65 \phantom{12} \\
 166.25 \\
 - 83.72 \\
 \hline
 82.53
 \end{array}$$

Make sure that the decimals line up.

Continue practise subtraction with large numbers and decimals throughout Year 5. If children are struggling with this method, then they should go back to the previous stage.

Year 6			
Objectives	Concrete	Pictorial	Abstract
solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why	There are no official objectives for Year 6 addition. However, throughout they should continue to add decimals and use the expanded addition method to add three and four digits to improve accuracy.		



solve problems involving addition, & subtraction	<p>Children should be encouraged to use mental methods and write jottings when needed. However, when it is not possible to calculate in their head, they should use the formal methods.</p> <p>There are no official objectives for Year 6 subtraction. However, throughout they should continue to use formal method for larger numbers and decimals and use these methods when solving problems. Children should be encouraged to use mental methods and write jottings when needed. However, when it is not possible to calculate in their head, they should use the formal methods.</p>
use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.	

## **Multiplication & Division**

### EYFS

Children learn through song, rhymes and practical activities. Through these activities, they will begin to solve problems that involve doubling.

#### **Multiplication**


By the end of Reception, children are expected to understand the concept of doubling and to be able to double a number up to 10. Before doubling can be introduced, children need to have a secure knowledge of counting, number facts and addition in order to double. Children are then introduced to the concept of doubling through practical games and activities, including the use of the outdoor areas. Children act out 'doubling' by physically add two equal groups together to find out the 'doubles' answer.



#### **Division**

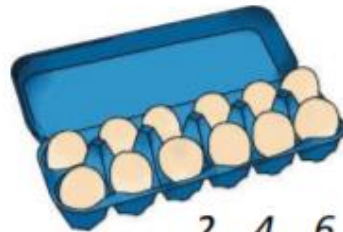
By the end of Reception, children are expected to understand the concept of halving and sharing. Before this can be introduced, children need to have a secure knowledge of counting backwards, number facts and subtraction in order to halve and share. Children are then introduced to the concept of halving and sharing through practical games and activities. They act out 'halving and sharing' through activities such as sharing food for their Teddy Bear's Picnic, sharing resources equally to play a game. This is reinforced by opportunities provided in the outdoor area for the children to halve and share out objects such as building blocks, twigs etc.



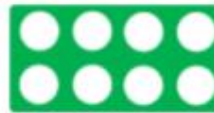
Early learning goals	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> <li>Solve problems, including doubling and halving.</li> <li>Y Solve practical problems that involve combining groups of 2, 5 or 10</li> </ul>	<p><b><u>Multiplication</u></b></p> <p>Where possible, concepts should be taught in the context of real life. Counting in repeated groups of the same size using real-life contexts and practical apparatus.</p> <div data-bbox="495 884 1288 1002">  <p>2... 4... 6 socks</p> </div> <p>Y Use pictorial representations alongside real objects (e.g. pairs of socks on a line or wellies on a rack). Y Sing, count and chant in twos, fives and tens (with and without objects).</p> <p>Use of the 100 square to identify patterns.</p>		

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

E.g. real eggs in a box, Numicon pieces, Numicon peg board, sorting trays.



2... 4... 6... 8... 10... 12 eggs



Two, four, six, eight



Ten, twenty, thirty

Y Start to combine groups by counting on from a set. E.g. Let's put 4 in our heads and carry on counting in twos. 4, 6, 8, 10.



Four, six, eight, ten

Solve practical problems that involve counting in twos, fives and tens.

E.g.



*How many wheels are there altogether?*

Understand doubling as adding the same number.



*Double 2 is 4*



*Double 3 is 6*



Y Begin to recall doubles and halves using songs and games.


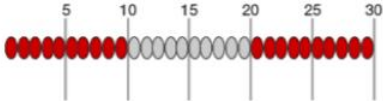

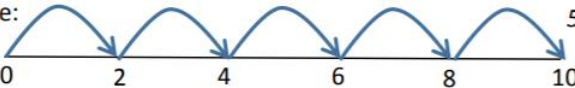
You can record calculations as pictures: numicon pieces can be drawn, counters, cubes etc.

### Division



Share the apples between two people. 'Half of the apples are for you and half are for me.'

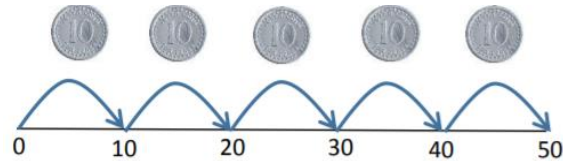


Year 1			
Nation curriculum objectives	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> <li>count in multiples of twos, fives and tens</li> <li>solve one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.</li> </ul>	<p><b><u>Multiplication</u></b></p> <p>Counting in twos, fives and tens</p>   <p>Understand multiplication in terms of counting groups of the same size (repeated addition)</p> <ul style="list-style-type: none"> <li>Use of concrete apparatus (e.g. washing line and other practical resources for counting, Numicon, bundles of straws, bead strings) and pictorial representations.</li> </ul> <p>E.g. There are 5 pairs of socks. How many socks are there altogether?</p>  <div data-bbox="1301 826 1603 986" style="border: 1px solid black; padding: 5px;"> <math>2 + 2 + 2 + 2 + 2 = 10</math>  or 5 lots of 2 equals 10 </div> <p>Record as jumps on a number line:</p>  <p><i>5 hops of 2 equals 10</i></p>		



Three pots of ten crayons. How many crayons altogether? 10,20, 30.

Sam has five 10p coins. How much does Sam have in total?



$10 + 10 + 10 + 10 + 10 = 50p$  or 5 lots of 10p equals 50p  
5 hops of 10 equals 50

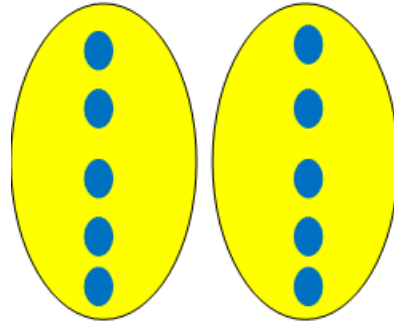
Using arrays and Numicon to understand multiplication.

Use arrays to support early multiplication. Arrays should be presented in both ways (two rows of five and five rows of twos) so that the children understand that the answer is the same.



Five groups of two faces. How many faces altogether? 2,4,6,8,10

Two groups of five faces. How many altogether? 5,10



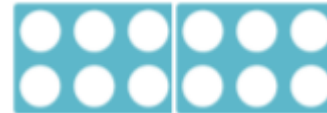
2 groups of 5. How many altogether?  $5 + 5 = 10$  Double five is ten

Continue to solve problems using practical contexts and develop vocabulary linked with multiplication throughout stage including writing number sentences.

#### Doubling numbers to 10 progressing to doubling numbers to 30



Double 3 =  $3 + 3$   
3 = 2 lots of 3



Double 6 =  $6 + 6 = 2$  lots of 6

#### Language of multiplication.

Use concrete apparatus to develop the vocabulary relating to 'times'. E.g. Pick up five, 4 times.



### Division

Children will start with practical activities that include sharing items. They need to share resources into equal groups. They will begin to use vocabulary associated with division.

For example, share these apples between two people, how many apples will each person have?



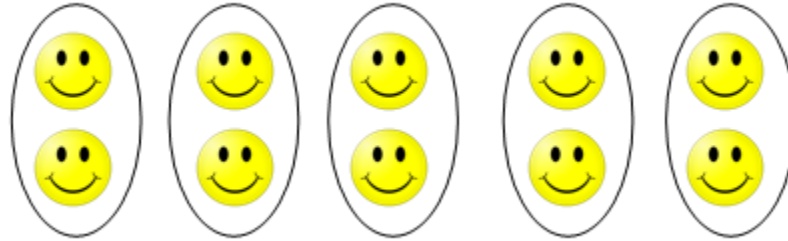
Share 20 crayons between 2 pots. How many crayons are there in each pot?



Then the children need to progress from sharing to grouping. You should use arrays to support division in the early stages.

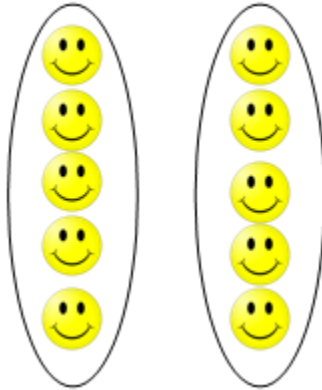


How many faces are there altogether? How many groups of 2?



There are five groups of two.

How many groups of five are there?



10 split into two groups is five. Half of ten is five

Year 2			
Nation curriculum objectives	Concrete	Pictorial	Abstract

- count in steps of 2, 3, and 5 from 0, and in tens from any number, forward and backward
- recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers
- calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs
- show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot
- solve problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and

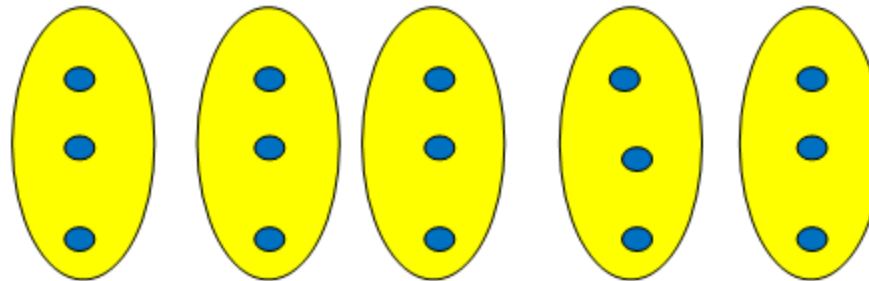
### **Multiplication**

Make sure that children are confident with the methods in the previous stage. Children need to continue to use a range of vocabulary to describe multiplication and use practical resources, pictures and diagrams.

Combining Groups (repeated addition)



If there are 3 groups of 10 crayons. How many crayons are there altogether?  $10+10+10=30$ . 3 groups of 10 is 3 times 10.  $3 \times 10=30$ ,  $10 \times 3 =30$



5 groups of 3, 5 lots of 3,  $3+3+3+3+3=15$   
5 times 3, 3 multiplied by 5,  $5 \times 3=15$ ,  $3 \times 5=15$

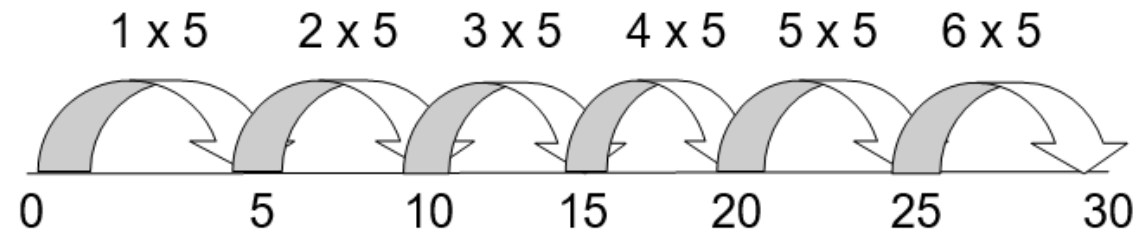
Using arrays to support multiplication:  
 $6 \times 5 =30$

multiplication and  
division facts, including  
problems in contexts.

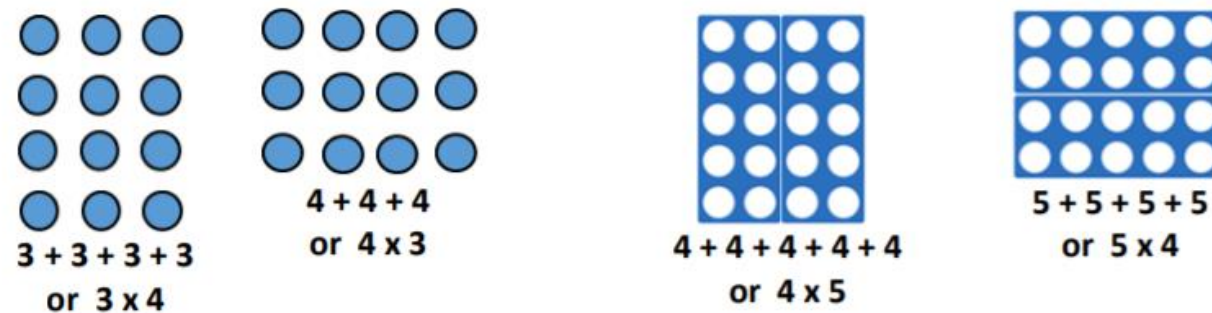


6 rows of 5. 5 groups of 6.  $5 \times 6 = 30$ ,  $6 \times 5 = 30$

Use an empty number line  
 $6 \times 5 = 30$



Use arrays to understand that multiplication can be done in any order (commutative).



Develop fluency in mental recall of the 2, 5- and 10-times tables. Use a variety of multi-sensory approaches to aid memorisation of times table facts: verbal repetition through songs and actions; use of images to support understanding; playing games (e.g. 'Fizz, buzz'); investigate patterns in the 100 square.

Strike out the multiples of 2 and circle the multiples of 5. What do you notice?

1	<del>2</del>	3	<del>4</del>	5	<del>6</del>	7	<del>8</del>	9	<del>10</del>
11	<del>12</del>	13	<del>14</del>	15	<del>16</del>	17	<del>18</del>	19	<del>20</del>
21	<del>22</del>	23	<del>24</del>	25	<del>26</del>	27	<del>28</del>	29	<del>30</del>

Recall doubles to 10 and understand doubling as multiplying by 2.



Begin to understand the effect of multiplying by 10 on place value. E.g. In  $4 \times 10$  the digit moves one place to the left. The zero is the place value holder.

Tens	Ones
	4
4	0

Make the connection to repeated addition. If the children are struggling then return to the previous stage.

### Division

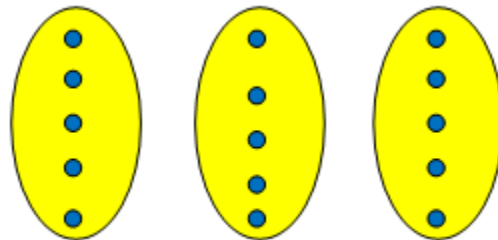
It is important that the children are confident with the previous method before moving on to the next stage.



30 crayons split between three groups. We have three pots and put ten crayons in each pot. How many pots do we need?  $30 \div 10 = 3$ .  $30 \div 3 = 10$ .

$$30 \div 10 = 3$$

$$30 \div 3 = 10$$



How many groups of 5? 15 shared equally between 3 people is....?

15 divided by 3 equals 5

15 divided by 5 equals 3

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$

Teacher should use arrays to support division.

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$



How many groups of 5?

How many groups of 3?

15 shared between 3 people is....?

15 shared between 5 people is.....?

$$15 \text{ divided by } 5 = 3$$

$$15 \text{ divided by } 3 = 5$$

When the children are ready, an empty number line can be used to count forwards.

$$30 \div 5 = 6$$

How many jumps make 30?

1 jump of 5    2 jumps of 5    3 jumps of 5    4 jumps of 5    5 jumps of 5    6 jumps of 5

0      5      10      15      20      25      30

Children can also jump back to make the link with repeated subtraction.  
 $30 \div 5 = 6$   
 How many groups of 5?

-5      -5      -5      -5      -5      -5

0      5      10      15      20      25      30

Year 3			
National curriculum objectives	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> <li>count from 0 in multiples of 4, 8, 50 and 100</li> <li>recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables</li> <li>write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for</li> </ul>	Develop fluency in mental recall of the 3, 4- and 8-times tables. Use a variety of multi-sensory approaches to aid memorisation of times table facts (see Year 2).  <div style="display: flex; justify-content: space-around; align-items: center;"> <div style="text-align: center;"> <p>"3"</p> </div> <div style="text-align: center;"> <p>"6"</p> </div> <div style="text-align: center;"> <p>"9"</p> </div> <div style="text-align: center;"> <p>"12"</p> </div> </div>		
	Multiply by 4 and 8 respectively, by doubling, doubling again (x4) and doubling again (x8).  Mental strategies. Multiply two-digit numbers by one-digit numbers by partitioning.		

two-digit numbers times one-digit numbers, using mental and progressing to formal written methods

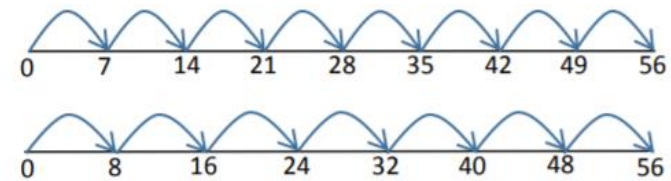
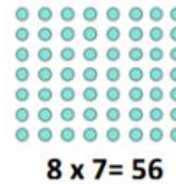
- solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects

E.g.  $13 \times 4 = 10 \times 4 + 3 \times 4 = 40 + 12 = 52$

$$\begin{array}{r} 13 \\ \times 4 \\ \hline 40 + 12 = 52 \end{array}$$

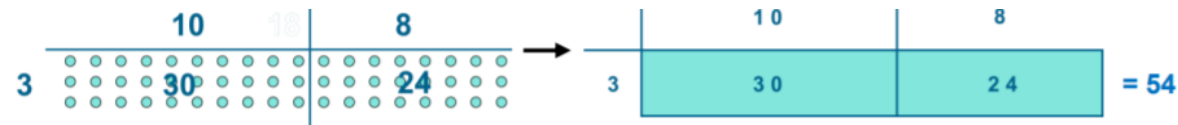
Continue to use number lines and arrays to support multiplication.

E.g.



**Begin to develop a written method.**

Some children will use arrays to begin to develop a written method (Grid Method) to multiply two-digit numbers by a single digit. E.g.  $18 \times 3$



**Partitioning alongside the grid method**

$13 \times 5 = 65$  (Partition 13 into  $10 + 3$ )

$10 \times 5 = 50$

$3 \times 5 = 15$

$50 + 15 = 65$

x	10	3
5	50	15

**Grid method**

$13 \times 8 = 104$

x	10	3
8	80	24

$80 + 24 = 104$

Partition 13 into  $10 + 3$  then multiply each number by 8. Add the partial products (80 and 24) together.

This will go into expanded short multiplication.

$$13 \times 8 = 104$$

$$\begin{array}{r} 10 + 3 \\ \times 8 \\ \hline 24 \text{ (3 x 8)} \\ + 80 \text{ (10 x 8)} \\ \hline 104 \end{array}$$

Remember to use an addition symbol to add the partial products together. Model the same calculation using a number line if necessary.

Formal short multiplication:

$$\begin{array}{r} 13 \\ \times 8 \\ \hline 104 \\ 2 \end{array}$$

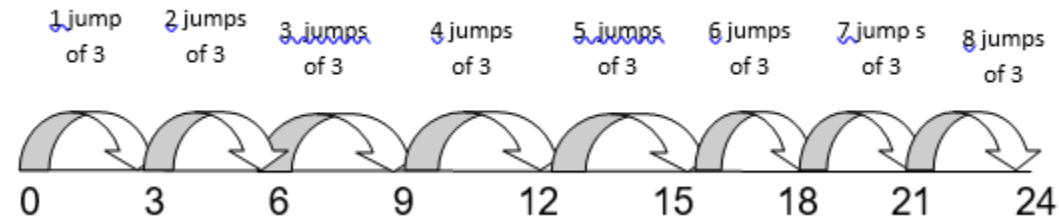
Make sure that the digit carried over is written under the line in the correct column. Continue to develop the formal method throughout Stage Three using two-digit numbers less than 20 multiplied by a one digit number. If children are confident they can multiply other two-digit numbers by a one digit. If children are making errors then return to the previous year.

### Division

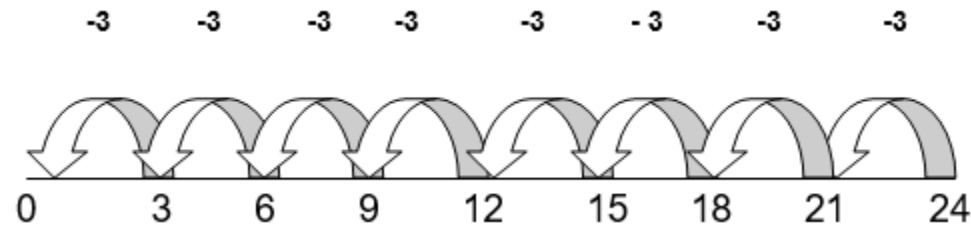
Children should continue to use practical resources and also, continue to use arrays, pictures, diagrams and number lines.

Continue to allow the children to use an empty number line to count forwards...

How many threes in 24?



Also jump back from 24 in 3s to show repeated subtraction.



How many groups of 3 in 24?

Then introduce the formal layout using multiplication and division facts that the children know:

$$24 \div 3 = 8$$

This can be displayed as:

$$3 \overline{) 24}$$

Twenty four divided by three equals eight.'

'How many threes are there in twenty four?'

Remember if the children are struggling with the method then take them back to the previous method.

Year 4			
National curriculum objectives	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> <li>count in multiples of 6, 7, 9, 25 and 1,000</li> <li>recall multiplication and division facts for multiplication tables up to <math>12 \times 12</math></li> <li>use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1;</li> </ul>	<p><b>Develop fluency in mental recall of multiplication facts up to <math>12 \times 12</math>.</b> Use a variety of multi-sensory approaches to aid memorisation of multiplication facts (see Year 2).</p> <p><b>Mental strategies.</b> Use known multiplication facts to derive associated facts. E.g. <math>8 \times 7 = 56</math>, so <math>80 \times 7 = 560</math> and <math>80 \times 70 = 5600</math>.</p> <p><b>Multiply three numbers together and use the associative law.</b> E.g. <math>4 \times 12 \times 5 = 4 \times 5 \times 12 = 20 \times 12 = 240</math></p> <p><b>Multiply two-digit numbers by one-digit numbers by partitioning.</b> E.g. deriving doubles. <math>37 \times 2 = 30 \times 2 + 7 \times 2 = 60 + 14 = 74</math></p>		



multiplying together 3 numbers

- recognise and use factor pairs and commutativity in mental calculations
- multiply two-digit and three-digit numbers by a one-digit number using formal written layout
- solve problems involving multiplying and adding, including using the distributive law to multiply two-digit numbers by 1 digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects

### Multiply one-digit and two-digit numbers by 10 and 100.

E.g.  $34 \times 100 = 3400$ . The zeros are the place value holders.

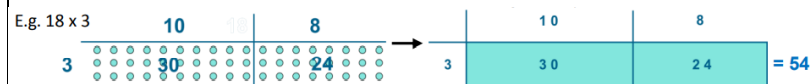
### Understand multiplication as scaling.

Solve practical problems where pupils need to scale up.

Relate to known number facts.

E.g. My sunflower is 25cm tall. Yours is 6 times taller. How tall is your sunflower?

Make sure that children are confident with the methods outlined in the previous year's guidance before moving on. Continue to use empty number lines and grid method.



Extend use of Grid Method to multiply three-digit numbers by one-digit numbers.

E.g.  $235 \times 6$

x	200	30	5
6	1200	180	30

$$\begin{array}{r} 1200 \\ + 180 \\ + 30 \\ \hline 1410 \\ 1 \end{array}$$

$$36 \times 4 = 144$$

$$120 + 24 = 144$$

x	30	6
4	120	24

Add the partial products.

Then move onto expanded short multiplication (two-digit number by a one digit number)

$$36 \times 4 = 144$$

1000s	100s	10s	1s
		3	4
3	4	0	0

$$\begin{array}{r}
 30 + 6 \\
 \times \quad 4 \\
 \hline
 24 \quad (4 \times 6 = 24) \\
 + 120 \quad (\underline{4} \times 30 = 120) \\
 \hline
 144
 \end{array}$$

Simplify the recording to:

$$\begin{array}{r}
 36 \\
 \times \quad 4 \\
 \hline
 + 24 \quad (4 \times 6) \\
 120 \quad (4 \times 30) \\
 \hline
 144
 \end{array}$$

This then leads onto short multiplication of a two-digit number multiplied by a one-digit number.  
 $36 \times 4 = 144$

$$\begin{array}{r}
 36 \\
 \times \quad 4 \\
 \hline
 144 \\
 \hline
 2
 \end{array}$$

Ensure that the digit carried over is written under the line in the correct column. Continue to practise the formal method of short multiplication of a two-digit number by a one-digit number throughout Year Four. If children are confident then develop to multiply by three-digit numbers multiplied by a one-digit number. If needed, you can return to the grid method or expanded method first.  
 $127 \times 6 = 762$

x	100	20	7
6	600	120	42

$600 + 120 + 42 = 762$

Expanded method

$$\begin{array}{r}
 127 \\
 \times 6 \\
 \hline
 42 \quad (6 \times 7) \\
 + 120 \quad (6 \times 20) \\
 + 600 \quad (6 \times 100) \\
 \hline
 762
 \end{array}$$

This will lead to short multiplication (formal method):

$$\begin{array}{r}
 127 \\
 \times 6 \\
 \hline
 762 \\
 \text{1} \quad \text{4}
 \end{array}$$

Ensure that place value language is used to ensure understanding and digits that carried over are written under the line in the correct column. If children are making significant errors, return to the previous year group.

### Division

Continue to use children's multiplication and division facts to solve calculations. If they are struggling, then take them back to the previous stage.

$$\begin{array}{r} 4 \\ 8 \overline{) 32} \end{array}$$

How many eights are there in 32? Continue the method, introducing remainders.

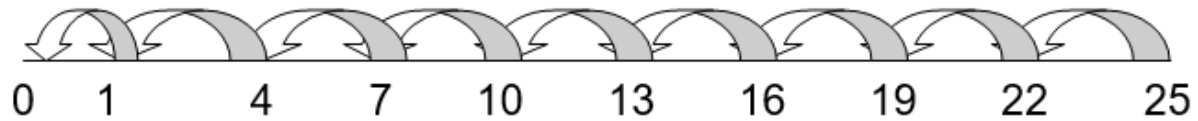
$$25 \div 3 = 8 \text{ r}1$$

$$\begin{array}{r} \text{8 r1} \\ 3 \overline{) 25} \end{array}$$

Remainders are not officially introduced until Stage 5 but it is a good opportunity to help them with their multiplication facts. It could be modelled using an empty number line.

Eight jumps of three and one left over.

$$25 \div 3 = 8 \text{ r}1$$



Alternatively you could jump forwards in multiples of three from zero to twenty four ('and one more makes 25')

Division using partitioning (two digits divided by one digit)

$$65 \div 5 = 13$$

$$65 = 50 + 15 \quad \text{Partition 65 into 50 and 15}$$

$$50 \div 5 = 10$$

$$15 \div 5 = 3$$

$$10 + 3 = 13$$

Continue to use empty number lines using multiples of the divisor.

$$65 \div 5 = 13$$

-15 (3x5)

-50 (10x5)



$$98 \div 7 = 14$$

$98 = 70 + 28$  Partition 98 into 70 and 28

$$70 \div 7 = 10$$

$$28 \div 7 = 4$$

$$10 + 4 = 14$$

This could be modelled on a number line for deeper understanding.

Children need to practise partitioning in many different ways.

$$\underline{98} \div 7 = 14$$

'We have partitioned 98 into 70 and 28 ( $90 = 70 + 28$ ).

Seven 'goes into' 70 ten times and seven 'goes into' 28 four times.

Ten add four equals 14'

$$10 + 4 = 14$$

$$7 \overline{) 70+28}$$

This will lead to the more formal method of division.

$$98 \div 7 = 14$$

$$14$$

$$7 \overline{) 928}$$

Continue to practise short division throughout Year 4. If you need to develop by dividing three digit by one digit using the formal method with whole numbers and no remainders. Remember to return to the previous stage if you are struggling.

Year 5

Year 5																																																																																		
National curriculum objectives	Concrete	Pictorial	Abstract																																																																															
<ul style="list-style-type: none"><li>identify multiples and factors, including finding all factor pairs of a number, and common factors of 2 numbers</li><li>know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers</li><li>establish whether a number up to 100 is prime and recall prime numbers up to 19</li><li>multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers</li><li>multiply and divide numbers mentally, drawing upon known facts</li><li>divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context</li></ul>	<b><u>Multiplication</u></b>																																																																																	
	Mental strategies Ensure fluent recall of multiplication facts up to $12 \times 12$ , and use confidently to derive associated facts. E.g. $8 \times 7 = 56$ , so $0.8 \times 7 = 5.6$ and $0.8 \times 0.7 = 0.56$ .																																																																																	
	Solve problems such as $15 \times 9$ using an efficient mental strategy. E.g. Partitioning: $15 \times 9 = (10 \times 9) + (5 \times 9) = 90 + 45 = 135$ ; multiply 15 by 10, then subtract 15.																																																																																	
	Multiply whole numbers and decimal numbers by 10, 100 and 1000. Use place value charts to support understanding (e.g. Gattengo chart)																																																																																	
	<table><tr><td>ten thousands</td><td>10 000</td><td>20 000</td><td>30 000</td><td>40 000</td><td>50 000</td><td>60 000</td><td>70 000</td><td>80 000</td><td>90 000</td></tr><tr><td>thousands</td><td>1000</td><td>2000</td><td>3000</td><td>4000</td><td>5000</td><td>6000</td><td>7000</td><td>8000</td><td>9000</td></tr><tr><td>hundreds</td><td>100</td><td>200</td><td>300</td><td>400</td><td>500</td><td>600</td><td>700</td><td>800</td><td>900</td></tr><tr><td>tens</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td><td>60</td><td>70</td><td>80</td><td>90</td></tr><tr><td>units</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td></tr><tr><td>tenths</td><td>0.1</td><td>0.2</td><td>0.3</td><td>0.4</td><td>0.5</td><td>0.6</td><td>0.7</td><td>0.8</td><td>0.9</td></tr><tr><td>hundredths</td><td>0.01</td><td>0.02</td><td>0.03</td><td>0.04</td><td>0.05</td><td>0.06</td><td>0.07</td><td>0.08</td><td>0.09</td></tr><tr><td>thousandths</td><td>0.001</td><td>0.002</td><td>0.003</td><td>0.004</td><td>0.005</td><td>0.006</td><td>0.007</td><td>0.008</td><td>0.009</td></tr></table>			ten thousands	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000	thousands	1000	2000	3000	4000	5000	6000	7000	8000	9000	hundreds	100	200	300	400	500	600	700	800	900	tens	10	20	30	40	50	60	70	80	90	units	1	2	3	4	5	6	7	8	9	tenths	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	hundredths	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	thousandths	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008
ten thousands	10 000	20 000	30 000	40 000	50 000	60 000	70 000	80 000	90 000																																																																									
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	Understand multiplication as scaling. Solve practical scaling problems (e.g. scaling up the dimensions of a model or building from a photograph).																																																																																	



- multiply and divide whole numbers and those involving decimals by 10, 100 and 1,000
- recognise and use square numbers and cube numbers, and the notation for squared ( $^2$ ) and cubed ( $^3$ )
- solve problems involving multiplication and division, including using their knowledge of factors and multiples, squares and cubes
- solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign
- solve problems involving multiplication and division, including scaling by simple fractions and problems involving simple rates

### Progress to formal written methods for multiplication

E.g.  $18 \times 13$

	10	8
10	100	80
3	30	24



		1	8		
	×	1	3		
		2	5	4	(3 × 18)
		1	8	0	(10 × 18)
		2	3	4	

1

Use estimation to check accuracy of calculations.

Make sure that the children are confident with the methods outlined in the previous year's guidance before moving on. When children are confident multiplying by one digit then move on to multiplying by two digits (less than 20)

$$23 \times 13 = (20+3) \times (10+3) = 299$$

x	20	3	
10	200	30	<b>230</b>
3	60	9	<b>+ 69</b>
			<b>299</b>

Add the partial products  $(200 + 30) + (60 + 9) = 299$

### Expanded long multiplication

$$\begin{array}{r}
 23 \\
 \times 13 \\
 \hline
 9 \quad (3 \times 3) \\
 60 \quad (3 \times 20) \\
 + 30 \quad (10 \times 3) \\
 200 \quad (10 \times 20) \\
 \hline
 299
 \end{array}$$

Then compacted long multiplication formal method:

$$23 \times 13 = 299$$

$$\begin{array}{r}
 23 \\
 \times 13 \\
 \hline
 + 69 \quad (3 \times 23) \\
 230 \quad (10 \times 23) \\
 \hline
 299
 \end{array}$$

Extend to larger two digit number:

$$56 \times 27 = (50 + 6) \times (20 + 7) = 1512$$

x	50	6	
20	1000	120	1120
7	350	42	392
			1512

Add the partial products  $(1000 + 120) + (350 + 42) = 1512$

Expanded long multiplication (two-digit numbers multiplied by two-digit numbers):

$$56 \times 27 = 1512$$

$$\begin{array}{r}
 56 \\
 \times 27 \\
 \hline
 42 \quad (7 \times 6) \\
 350 \quad (7 \times 50) \\
 + 120 \quad (20 \times 6) \\
 1000 \quad (20 \times 50) \\
 \hline
 1512
 \end{array}$$

This moves onto compact long multiplication.

$$56 \times 27 = 1512$$

$$\begin{array}{r}
 | \quad 56 \\
 \times 27 \\
 \hline
 392 \quad (7 \times 56) \\
 + 1120 \quad (20 \times 56) \\
 \hline
 1512
 \end{array}$$





The remainder can also be expressed as a fraction,  $\frac{2}{5}$  (the remainder divided by the divisor):

$$432 \div 5 = 86 \frac{2}{5}$$

$$\begin{array}{r} 086r2 \\ 5 \overline{)432} \\ \underline{40} \phantom{0} \\ 32 \phantom{0} \\ \underline{30} \phantom{0} \\ 20 \phantom{0} \\ \underline{20} \phantom{0} \\ 0 \end{array}$$

Continue to practise the formal method with remainders. Make sure that the children interpret the answer and present the answer in the right context including decimals.

$$\begin{array}{r} 086.4 \\ 5 \overline{)432.200} \\ \underline{40} \phantom{00} \\ 32 \phantom{00} \\ \underline{30} \phantom{00} \\ 20 \phantom{00} \\ \underline{20} \phantom{00} \\ 00 \end{array}$$

Year 6			
National curriculum objectives	Concrete	Pictorial	Abstract
<ul style="list-style-type: none"> <li>multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication</li> <li>divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by</li> </ul>	<p><b><u>Multiplication</u></b></p> <p>Mental strategies. Ensure fluent recall of multiplication facts up to <math>12 \times 12</math>, and derive associated facts (e.g. <math>0.8 \times 0.7 = 0.56</math>).</p> <p>Derive associated facts from known multiplication facts. E.g. If <math>126 \times 42 = 5292</math>, use the this fact to work out: <math>12.6 \times 42</math>; <math>12.6 \times 4.2</math>; <math>12.6 \times 4200</math>.</p> <p>Solve problems such as <math>15 \times 9</math> using an efficient mental strategy (see Year 5).</p> <p>Multiply whole numbers and decimal numbers by 10, 100 and 1000 (see Year 5).</p>		

rounding, as appropriate for the context

- divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- perform mental calculations, including with mixed operations and large numbers
- identify common factors, common multiples and prime numbers
- use their knowledge of the order of operations to carry out calculations involving the 4 operations
- solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why
- solve problems involving addition, subtraction, multiplication and division
- use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy

Identify common multiples of given numbers.

E.g. Find common multiples of 40 and 90.

Use BIDMAS to solve problems that use knowledge of the order of operations.

E.g. Understand how to solve  $7 + 4 \times 3$  and  $(7 + 4) \times 3$ .

Understand multiplication as scaling.

Solve practical scaling problems (e.g. scaling up the dimensions of a model or building from a photograph).

Formal written methods to multiply up to 4-digit numbers by 2-digit whole numbers.

Develop fluency and deepen understanding of the compact written methods of short and long multiplication. Use estimation to check answers to calculations.

E.g.  $567 \times 36$

Estimate =  $600 \times 40 = 24,000$

			5	6	7
×			3	6	
	3	<sup>4</sup> 4	<sup>4</sup> 0	2	
1	<sup>2</sup> 7	<sup>2</sup> 0	1	0	
<sup>1</sup> 2	0	4	1	2	

Extend to multiplying decimals (e.g.  $6.42 \times 32$ ).

Ensure that children are confident with the methods in the previous stages. Continue to practise the formal short and long multiplication method with larger numbers and decimals. Use the expanded method first if needed.

**The grid method** (decimal number multiplied by a two-digit number):

$53.2 \times 24 = 1276.8$

<u>x</u>	<u>50</u>	<u>3</u>	<u>0.2</u>	
20	1000	60	4	1064.0
4	200	12	0.8	212.8
				1276.8

The formal written method of long multiplication:

$$\begin{array}{r}
 \phantom{x} 3.2 \\
 x \phantom{0} 24.0 \\
 \hline
 212.8 \quad (53.2 \times 4) \\
 1064.0 \quad (53.2 \times 20) \\
 \hline
 1276.8
 \end{array}$$

It is an option to include .0 in this example, but not essential. The prompts can be taken away if children no longer needed them.

If children are making lots of errors then they should go back to the previous stage. By the end of Year 6 children should use mental methods when appropriate. They should use a formal method when they cannot do calculations in their heads.

### **Division**

Make sure that children are confident with the method before moving on. Continue to practise the formal method of division with and without remainders.

$$496 \div 11 = 45 \text{ r}1$$

The remainder can also be expressed as a fraction,  $\frac{1}{11}$  (the remainder divided by the divisor)

Dividing by two digit numbers using a formal method of long division.

$$\begin{array}{r}
 45 \text{ r}1 \\
 \hline
 11 \overline{) 496} \\
 \underline{- 440} \quad (40 \times 11) \\
 56 \\
 \underline{- 55} \quad (5 \times 11) \\
 \underline{1} \text{ (R)}
 \end{array}$$

Multiples of the divisor (11) have been subtracted from the dividend (496)

40 (lots of 11) + 5 (lots of 11) = 45  
(lots of 11)'

'1 is the remainder'

Answer: 45

Short division does not work to solve this calculation. However, long division will help solve it.

$$144 \div 16 = 9$$

$$\begin{array}{r}
 9 \\
 \hline
 16 \overline{) 144} \\
 - \quad 64 \text{ (4 x 16)} \\
 \hline
 80 \\
 - \quad 64 \text{ (4 x 16)} \\
 \hline
 16 \\
 - \quad 16 \text{ (1 x 16)} \\
 \hline
 0 \\
 16
 \end{array}$$

Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16)

There is no remainder

$$432 \div 15 = 28 \text{ r}12$$

$$\begin{array}{r}
 \text{28 r}12 \\
 \hline
 15 \overline{) 432} \\
 - \quad 300 \text{ (20 x 15)} \\
 \hline
 132 \\
 - \quad 120 \text{ (8 x 15)} \\
 \hline
 12 \text{ (remainder)}
 \end{array}$$

Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28

12 is the remainder



The remainder can also be expressed as a fraction  $\frac{12}{15}$  (the remainder divided by the divisor) or as a decimal, **0.8** (see next example)

The answer is: 28  $\frac{12}{15}$  or 28.8

This is an alternative way to teach division but they must be secure in the previous method before moving on to:

$$\begin{array}{r}
 432.0 \\
 30 \downarrow \\
 132 \\
 120 \downarrow \\
 120 \\
 \hline
 120 \\
 \hline
 0
 \end{array}$$

$$432 \div 15 = 28.8$$

The remainder should be expressed as a decimal.

Children should be using mental methods and jotting when appropriate. When they are unable to do the calculation in their head then they should use a formal written method.